

Chapter 6

Growth and Ideas

Learning Objectives:

In this chapter you will learn

- That new ideas — new ways of using existing resources — are the key to sustained long-run growth.
- The meaning of “nonrivalry,” and why this property makes ideas differ from other economic goods in a crucial way.
- How the economics of ideas involves increasing returns and problems with Adam Smith’s invisible hand.
- A new model of economic growth: the Romer model.
- How to combine the Romer model and the Solow model to get a full theory of long-run economic performance.

Every generation has perceived the limits to growth that finite resources and undesirable side effects would pose if no new recipes or ideas were discovered. And every generation has underestimated the potential for finding new recipes and ideas. We consistently fail to grasp how many ideas remain to be discovered.

— Paul Romer, “Economic Growth,” *The Fortune Encyclopedia of Economics*, David R. Henderson (ed.), New York: Time Warner Books, 1993.

6.1 Introduction

One of the early themes of this book is that all models abstract from most features of the world in order to highlight a few crucial economic concepts. The Solow model, for example, draws a sharp distinction between capital and labor and focuses on capital accumulation as a possible engine of economic growth. What we saw in the previous chapter is that this model leads to numerous insights but ultimately fails to provide a theory of sustained growth in income.

In a very famous paper published in 1990, Paul Romer suggested that an even more fundamental distinction should play a crucial role in the study of economic growth. In particular, Romer divided the world of economic goods into *objects* and *ideas*.¹

Objects include most goods that we are familiar with. Capital and labor from the Solow model are objects, as are land, cell phones, oil, jet planes, computers, pencils, and paper.

Ideas, on the other hand, are instructions or recipes. Ideas include designs for making objects. For example, the set of instructions for turning sand — silicon dioxide — into computer chips is an idea. Before the invention of this idea, sand had value to beach-goers, kids with shovels, and glass-blowers. But with the discovery of the

¹Romer’s original paper is a classic, although the mathematics after the first several pages is challenging: Paul M. Romer, “Endogenous Technological Change” *Journal of Political Economy*, Volume 98, October 1990, pp. S71-S102.

recipe for converting sand into computer chips, a new and especially productive use of sand was created. Other ideas include the design of a cell phone or jet engine, the manufacturing technique for turning petroleum into plastic, and the set of instructions for changing trees into paper.

Ideas need not be confined to feats of engineering, however. The management techniques that make WalMart the largest private employer in the United States are ideas. So are the just-in-time inventory methods of Japanese automakers and the quadratic formula of algebra.

Dividing economic goods into objects and ideas and thinking carefully about the implications of this division leads to the modern theory of economic growth. This theory turns out to have wide-ranging implications for many areas of economics, including the economics of intellectual property, anti-trust economics, international trade, and economic development. This “idea about ideas” is one of the most important contributions of economics during the last two decades of the twentieth century.

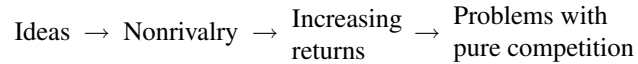
The first part of this chapter provides an overview of the economics of ideas, developing a number of key insights in the process. Next, we present a simple model of idea-based economic growth that exploits these insights. Finally, we show how this model can be combined with the Solow model to generate a rich theory of long-run economic performance.

6.2 The Economics of Ideas

Romer’s division of goods into objects and ideas led him to think carefully about how the economics of ideas may differ from the economics of objects. The economics of objects has been studied for centuries. It forms the basis of Adam Smith’s invisible hand result that perfectly competitive markets lead to the best of all possible worlds. The economics of ideas turns out to be different, and these differences are what make sustained economic growth possible.²

²While Paul Romer took the most important step in developing idea-based growth theory, many other researchers also share credit. In the 1960s, Kenneth Arrow, Zvi Griliches, Dale Jorgenson, William Nordhaus, Edmund Phelps, Karl Shell, Hirofumi Uzawa, and many others made substantial progress. Important advances following Romer’s work have been made by Philippe Aghion, Robert Barro, Gene Grossman, Elhanan Helpman, Peter Howitt, Robert Lucas, Martin Weitzman, and others.

As a guide to our discussion, consider the following *Idea Diagram*:



We will discuss each of the elements in this diagram in turn.

6.2.1 Ideas

One way of viewing the distinction between objects and ideas is that objects are the raw materials of the universe — atoms of carbon, oxygen, silicon, iron, etc. — and ideas are instructions for using these atoms in different ways. One arrangement of these raw materials yields a diamond, another yields a computer chip. One yields a powerful new antibiotic, and one yields the manuscript for Einstein’s theory of special relativity. New ideas are new ways of arranging raw materials in ways that are economically useful.

How many potential ideas are there? Suppose we limit ourselves to instructions that can be written in a single paragraph of 100 words or less, about the length of the abstract to most scientific papers. For example, consider the germ theory of disease, demonstrated by Louis Pasteur in the middle of the 19th century: “Microorganisms called germs are responsible for many diseases. Good sanitation in the household and on the surgeon’s table can save many lives.” Or consider the electric dynamo for converting mechanical energy into electricity: “A spinning magnet inside an iron ring will induce an electric current in a wire that spirals around the ring.”

The English language contains more than 20,000 words. How many different idea paragraphs can we create? With 100 words per paragraph, the answer is $(20,000)^{100}$ which is larger than 10^{430} , or a one followed by 430 zeros. Of course most of these word combinations will be complete gibberish, but some paragraphs will describe the Fundamental Theorem of Calculus, a summary of Darwin’s theory of evolution, the chemical formula for penicillin, a description of the double helix structure of DNA, and perhaps even a warp drive to power spaceships in the future. To put this huge number into context, suppose only one in 10^{100} of these paragraphs contains a potentially useful and coherent idea. That would still leave 10^{330} possible idea paragraphs, which is gazillions of times larger than the number of particles in the known universe.³

³Scientists guesstimate that there are on the order of 4×10^{77} particles in the universe.

The amount of raw material in the universe — the amount of sand, oil, and the number of atoms of carbon, oxygen, etc. — is finite. But the number of ways of arranging and using these raw materials is so large as to be virtually infinite. Economic growth occurs as we discover better and better ways to use the finite resources available to us. That is, sustained economic growth occurs because we discover new ideas.

6.2.2 Nonrivalry

Consider the next word in our Idea Diagram, “nonrivalry.” Objects, such as cellphones or chalkboards or professors, are rivalrous. That is, one person’s use of a particular object reduces the inherent usefulness of the object to another person. If you are using your cellphone to make a call, I cannot use it. If the economics class is using a particular chalkboard at a particular time, the mathematicians cannot use it simultaneously. Most goods in economics are rivalrous, and it is this characteristic that gives rise to the scarcity that is the central subject of study in economics.

This notion that objects are rivalrous is so natural that it barely needs explaining. However, it comes into sharp relief when compared to the fact that ideas are nonrivalrous.

My use of an idea does not inherently reduce the “amount” of the idea available for you to use. The quadratic formula is not itself scarce, and the fact that I am using the formula to solve an equation does not make it any less available for your use. Symphonies around the world can perform Mozart’s *Magic Flute* simultaneously: once the opera has been composed, one symphony’s performance does not make the composition itself more scarce in any sense.

Because nonrivalry is probably a new concept to you, let’s go through an example more carefully. Consider the difference between the design of a computer and the computer itself. The computer is certainly rivalrous: if you are using particular CPU cycles to browse your favorite web site, those cycles cannot be used by me to listen to my favorite song or by our friend to estimate an econometric model of stock prices. One person’s use of the computer reduces the potential benefit to other people from using the computer.

The design for the computer is different, however. Suppose we have a factory in

Taiwan that uses a particular design for producing a computer. The factory has 27 assembly lines running fulltime to produce the latest laptop. Notice that each assembly line can use the same design. We do not need to invent a new design for each assembly line, and if we wish to add another assembly line, we just follow the same set of instructions for putting together the computer. The design for the computer does not have to be reinvented for each production line. As an idea, the design is nonrivalrous: it can be used by any number of people without reducing the inherent usefulness of the idea.⁴

We should be careful with this concept of scarcity. New ideas surely are scarce: it would always be nice to have faster computers or better batteries or improved medical treatments. But existing ideas are not inherently scarce themselves. Once an idea has been invented, it can be used by an arbitrary number of people without anyone's use being degraded.

6.2.3 Increasing Returns

The fact that particular designs and instructions are not scarce in the same way that objects are scarce is the first clue that the economics of ideas is different from the economics of objects. This becomes clear in the next link in the Idea Diagram, to “increasing returns.”

Consider the production of a new pharmaceutical, such as a new antibiotic. Coming up with the precise chemical formula and manufacturing technique for the new antibiotic is the hard part. Indeed, current estimates suggest that the average cost of developing a new drug is on the order of \$800 million.⁵

Once the new antibiotic has been invented, though, it is reasonable to think of production as occurring with a standard constant returns to scale production function. After all, doses of the antibiotic are just some *object* and we have already discussed the production of objects in the previous two chapters. In this case, suppose a given

⁴If each assembly line requires its own physical set of blue prints, that is fine. The blueprints from another line can be photocopied. The paper they are printed on is a rivalrous object. But the design itself is the idea and it is nonrivalrous.

⁵For some interesting discussion of this number see the recent book review by John P. Moore in the *Journal of Clinical Investigation* Volume 114, 2004, p. 1182 of Merrill Goozner's book entitled *The \$800 million pill: the truth behind the cost of new drugs*.

factory with a given workforce and raw materials for inputs can produce 100 doses of the antibiotic per day. If we wish to double the daily production of the antibiotic, we can simply build an identical factory, use an identical collection of workers, and purchase the same quantity of the various materials used in production. Doubling all of these inputs will exactly double production. This is the *standard replication argument* we used in previous chapters. If each of the first 100 doses costs \$10 to produce, then each of the second 100 doses will also cost \$10 to produce.

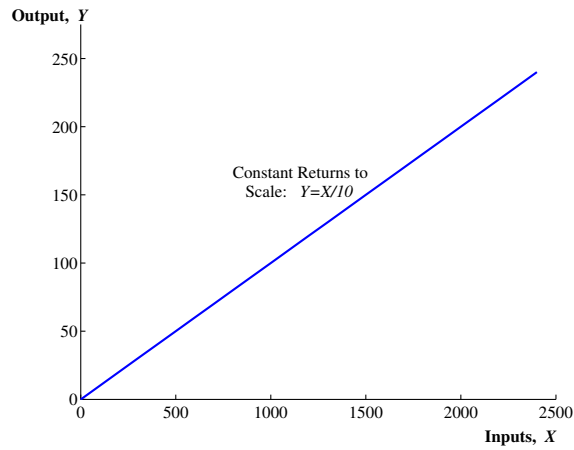
But now consider the entire chain of production, starting from the invention of the new antibiotic. The first \$800 million is used to create the instructions for making the antibiotic, producing no actual doses of the drug. To get one dose, we spend \$800 million for the design plus \$10 in manufacturing costs. After that, if we then spend another \$800 million, we produce 80 million doses. Doubling inputs leads to much more than a doubling of outputs. Therefore, the production function is characterized by increasing returns to scale once we include the fixed cost of creating the drug in the first place.

Figure 6.1 illustrates this antibiotic example graphically. Panel (a) of the figure shows the constant returns to scale production function for producing the antibiotic after the chemical formula has been created. For each \$10 spent, one dose of the drug is produced. Notice that average production per dollar spent, Y/X , is constant; it does not vary with the scale of production. Doubling the inputs exactly doubles the output.

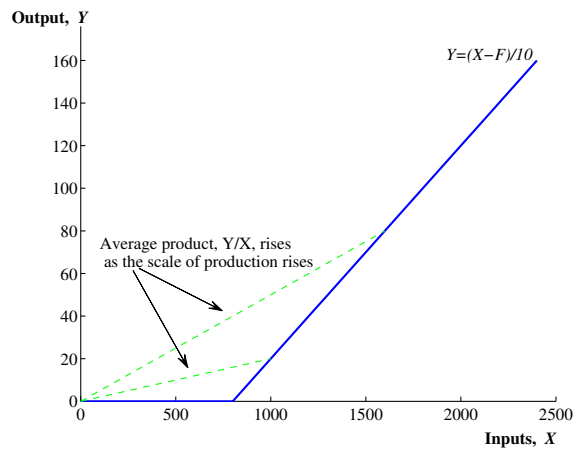
Panel (b) of the figure shows this same production function, but now including the fixed cost of $\bar{z} = \$800$ million that must be paid before any of the antibiotic can be produced. If X denotes the amount of money spent producing the antibiotic, the production function is $Y = (X - \bar{z})/10$, once X is larger than \bar{z} . In this case, the average production per dollar spent, Y/X , is increasing as the scale of production rises. This can be seen in the graph, or also by noting that $Y/X = (1 - \bar{z}/X)/10$, which is increasing in X .

We can be more precise in our discussion of increasing returns by going back to the standard production function we've used in previous chapters. Suppose output Y is produced using capital K and labor L . But suppose there is also another input into production, called "knowledge" or the stock of ideas. Denote this stock of ideas by A .

Figure 6.1: How a Fixed Cost Leads to Increasing Returns: The Antibiotic Example



(a) Constant Returns to Scale: $Y = X/10$



(b) Increasing Returns from Fixed Cost $\bar{z} = 800$

Panel (a) shows a constant returns to scale production function, $Y = X/10$. Notice that the average product Y/X is constant. Panel (b) shows the same production function, but this time with an additional fixed cost \bar{z} of \$800 million that must be paid before production can occur. This leads to increasing returns to scale. One way to see this is to notice that the average product of the input, Y/X , now increases as the scale of production rises.

As in previous chapters, let our production function be

$$Y_t = F(K_t, L_t, A_t) = A_t K_t^{1/3} L_t^{2/3}. \quad (6.1)$$

The only difference between this production function and the one we have used earlier is that we have replaced our TFP parameter \bar{A} with the stock of ideas, A_t . We have given it a new name and a time subscript.

First, notice that this production function exhibits constant returns to scale in K and L . If we wish to double the amount of output produced in this economy, we can just double the amount of capital and the amount of labor used. This is the standard replication argument: we just replicate the factory exactly as it is (including the workers, materials, etc.) in order to double output.

Now, though, there is an added twist because the stock of knowledge enters the production function. If we think of this as the production function for a new antibiotic, we can interpret it as follows. To double the production of the antibiotic, we only need to double the “objects” involved in production. In particular, because the chemical formula for the antibiotic is nonrivalrous, it can be used by *both* of the factories we set up. We certainly do not need to reinvent the chemical formula for the new factory. Every worker who needs it can use the chemical formula that has been discovered.

Notice what this implies about the returns to scale to all of the inputs in production, both objects and ideas. If we double capital, labor, *and* knowledge, we will more than double the amount of output produced:

$$F(2K, 2L, 2A) = 2A(2K)^{1/3}(2L)^{2/3} = 2 \cdot 2^{1/3} \cdot 2^{2/3} \cdot AK^{1/3}L^{2/3} = 4 \cdot AK^{1/3}L^{2/3} = 4 \cdot F(K, L, A).$$

Since doubling inputs more than doubles output, this production function exhibits increasing returns to ideas and objects taken together.

Increasing returns is one of the crucial implications of the economics of ideas, and despite all of the algebra in this section, the reasoning is relatively straightforward. So let’s try to summarize it in a few sentences. Motivated by the standard replication argument, there are constant returns to objects in production. To double the production of any good, we simply replicate the objects that are currently used in production; the same stock of knowledge can be used since ideas are nonrivalrous. This necessarily

then implies that there are increasing returns to both objects and ideas: if doubling the objects is enough to double production, then doubling the objects and the stock of knowledge will more than double production.

6.2.4 Problems with Pure Competition

The last link in our Idea Diagram suggests that the increasing returns generated by nonrivalry leads to problems with pure competition. What are these problems?

To begin, recall the beauty of Adam Smith's invisible hand theorem. Under the assumption of perfect competition, markets lead to an allocation that is Pareto optimal. That is, there is no way to change the allocation of resources to make someone better off without making someone else worse off. In this sense, markets produce the best of all possible worlds.⁶

Perfectly competitive markets achieve this optimal allocation by equating marginal costs and marginal benefits through a price system. An essential part of the story is that price equals marginal cost in all markets. And this is where the problem occurs when there are increasing returns to scale.

Going back to our antibiotic example, what would happen if the pharmaceutical company were forced to charge a price equal to marginal cost? At first, it appears nothing goes wrong. The marginal cost of producing a dose is \$10, and if the firm sells the antibiotic at \$10, it just breaks even, leading to one of the hallmarks of perfect competition, zero profits.

But suppose we go back one stage earlier. Suppose the pharmaceutical company has not yet invented the new drug. Will it undertake the \$800 million research effort in order to discover the chemical formula for the new antibiotic? If it does, it sinks \$800 million dollars, discovers the formula, and then sells the drug at marginal cost. Including the original research expenditures, the firm loses \$800 million. *So if prices are equal to marginal cost, no firm will undertake the costly research that is necessary to invent new ideas.* Pharmaceuticals must sell at a price greater than marginal cost in

⁶The invisible hand theorem is also known as the First Fundamental Theorem of Welfare economics. A limitation of the theorem is that it says nothing about equity. For example, you having everything in the economy is Pareto optimal: we can't make me better off without making you worse off.

order to allow the producer to recoup the original research expenditures that led to the discovery in the first place.

This point is obviously much more general than the antibiotics example. Any time new ideas are invented, there is a fixed cost to produce the new set of instructions. After that, production proceeds with constant returns to scale and constant marginal cost. But in order for the innovator to be compensated for the original research that led to the new idea, there must be some wedge between price and marginal cost at some point down the line. This is true for drugs, computer software, music, computer operating systems, cellphones, jet airplanes, and even economics textbooks. One of the main reasons new goods get invented is because of the incentives embedded in the wedge between price and marginal cost.

This wedge means that markets cannot be characterized by pure competition if we are to have innovation. This is one justification for the patent and copyright systems. Patents reward innovators with monopoly power for 20 years in exchange for the inventor making the knowledge underlying the discovery public. This monopoly power provides a temporary wedge between price and marginal cost that leads to profits. These profits, in turn, provide the original incentive for the innovator to seek out the new idea in the first place.

———— BOX: What about Open Source Software and Altruism? ————

Profits are not the only incentive for people to create new ideas. One of the more interesting alternatives is the Open Source movement in computer software. Linux (a computer operating system) and Apache (one of the most popular pieces of software for running web sites) are examples of sophisticated computer software that are available for free — equal to the zero marginal cost of production.

How are the large research costs of creating this software financed, if not from the subsequent profits associated with a price greater than marginal cost? The answer is that many people willingly spend their free time writing and improving these computer programs. Some financing does come from existing companies, but to a great extent volunteers support this software. Feelings of altruism may be part of the motivation, as is a desire to show off programming skills to other people, including potential employ-

ers or venture capitalists.⁷

———— End of Box ————

How can we best provide the appropriate incentives for innovation? Patents and copyrights are one approach. Trade secrets — where the details of a particular idea are withheld from competitors — is another approach for generating a wedge between price and marginal cost; this approach is quite common in industries like financial services and retailing, where patents are uncommon.

Incentives for innovation that require prices greater than marginal cost have an important negative consequence. Consider a pharmaceutical company that invests a billion dollars in developing a new cancer drug. Suppose the marginal cost of producing the drug is only a hundred dollars for a year's worth of treatments. For the reasons we have just discussed, the drug company may charge a price much greater than marginal cost — say \$10,000 per year — for treatment. The additional revenue goes to cover the research cost of the drug. However, there will always be people who could afford the drug at the marginal cost of \$1000, but who cannot afford the drug at the monopoly price of \$10,000. These people are priced out of the market, and this results in a (potentially large) loss in welfare.

Other approaches may avoid the distortion associated with prices that are above marginal costs. For example, governments provide incentives for research by using tax revenue to fund research. Successful examples of this approach include the National Science Foundation, the National Institutes of Health, and the Department of Defense funding of the ArpaNet, a precursor to today's World Wide Web.

Prizes provide yet another alternative. For example, in the 1920s, hotel magnate Raymond Orteig offered a \$25,000 prize for the first nonstop flight between New York and Paris. Charles Lindbergh and his famous plane "The Spirit of St. Louis" captured this prize in 1927, and the prize is credited with spurring progress in aviation. More recently, the \$10 million Ansari X Prize has had a similar effect on private space ventures. Michael Kremer of Harvard University has even proposed that development

⁷See Josh Lerner and Jean Tirole, "Some Simple Economics of Open Source," *Journal of Industrial Economics*, 2002, Volume 50, pp. 197-234.

organizations fund large prizes as a way to spur innovation in creating vaccines for AIDS and malaria in developing countries.⁸

Which of these mechanisms — patents, trade secrets, government funding, or prizes — provide the best incentives for innovation and maximizes welfare? Of course, these mechanisms are themselves ideas that were invented at some point. Patents, for example, first appeared in England in the 17th century, although with limited application and enforcement. It seems likely, then, that we have not yet discovered the best mechanisms for providing the right incentives for innovation. Such “meta-ideas” may be among the most valuable discoveries we can make.⁹

6.3 The Romer Model

This discussion of the economics of ideas leads directly to the key insights we need in order to get a model of sustained growth in per capita income. In particular, we need a model that emphasizes the distinction between ideas and objects and, because of the nonrivalry of ideas, this model must incorporate increasing returns.

Let’s describe the model verbally before getting into the mathematical details. Fortunately, the Romer model turns out to be even simpler than the Solow model, so this is relatively easy.

In the Romer model, there are two key goods that are produced: a consumption object called “output” and new ideas. Output and new ideas are both produced in the same way, by combining workers and existing ideas. Ideas cumulate over time, the way capital did in the Solow model. However, because they are nonrivalrous, the accumulation of ideas is able to sustain growth in a way that the accumulation of capital could not. And that’s it!

We will follow Romer’s logic and emphasize the distinction between ideas and objects in our model. We will downplay Solow’s distinction between capital and labor in order to bring out the key role played by ideas. In fact, in the main model that follows,

⁸See <http://www.ksg.harvard.edu/ksgnews/KSGInsight/kremer.htm>.

⁹For some interesting thoughts on this question see Michael Kremer, “Patent Buyouts: A Mechanism for Encouraging Innovation,” *Quarterly Journal of Economics*, November 1998, pp. 1137–1167; also Suzanne Scotchmer, *Innovation and Incentives* (MIT Press, 2004).

we omit capital completely to keep things simple. Section 6.14 will reintroduce capital, so don't let this omission bother you.

To translate Romer's story into a mathematical model, consider first the production functions for the consumption good and for new ideas:

$$Y_t = A_t L_{yt} \quad (6.2)$$

$$\Delta A_t = \bar{z} A_t L_{at} \quad (6.3)$$

The first equation is the production function for output, Y_t . Output is produced using the stock of knowledge, A_t , and labor, L_{yt} . Notice that this production function features all of the key properties discussed in the previous section. In particular, there are constant returns to objects in the production function. If we wish to double the production of output, we simply double the number of workers. Because ideas are nonrivalrous, the new workers can use the same stock of ideas. But knowledge is also used in the production of output, and there are therefore increasing returns to ideas and objects in this production function.

The second equation, equation (6.3), is the production function for new ideas. A_t is the stock of ideas at time t . Recall that Δ is the "change over time" operator, so that $\Delta A_t \equiv A_{t+1} - A_t$ is the change in the stock of ideas. But that is just another way of saying ΔA_t is the number of new ideas produced during period t . Our second equation, then, says that new ideas are produced using existing ideas A_t and workers L_{at} . The only difference between our production function for output and our production function for ideas is the presence of a productivity parameter \bar{z} in the idea production function. This allows us to study experiments in which the economy gets better at producing ideas. We also assume the economy starts out at date $t = 0$ with an existing stock of ideas, \bar{A}_0 .

Notice that it is the same stock of ideas that gets used in both the production of output and the production of new ideas. Again this is because ideas are nonrivalrous: they can be used by many people for many different purposes simultaneously. In contrast, workers are an object. If a worker spends her time producing automobiles, that time is used up and cannot simultaneously be used to conduct research on new antibiotics. In

our model, this shows up as a resource constraint:

$$L_{yt} + L_{at} = \bar{N}. \quad (6.4)$$

That is, the number of workers producing output and the number of workers producing ideas add up to the total population, \bar{N} , which we take to be a constant parameter.

Let's pause now to count equations and endogenous variables. We have three equations at this point: the two production functions and the resource constraint. As for endogenous variables, we have Y_t , A_t , L_{yt} , and L_{at} . (Recall that ΔA_t is just a function of A_t , so we do not need to count it separately.) We need one more equation to close the model.

Think about the economics of the model and ask yourself what is missing (the answer comes in the next sentence, so pause here if you want to figure it out for yourself). The answer is that we need an equation that describes how labor gets allocated to its two uses. What determines how much labor works to produce output versus ideas?

Here is where we make a useful simplification. In Romer's original model, he set up markets for labor and output, introduced patents and monopoly power to deal with increasing returns, and let the markets determine the allocation of labor between producing goods and producing new ideas. What Romer found is fascinating and you may have already figured it out. In particular, Romer found that unregulated markets in this world do *not* lead to the best of all possible worlds. There is a tendency for markets to provide too little innovation relative to what is optimal. In the presence of increasing returns, Adam Smith's invisible hand may fail to get things right.¹⁰

Going through the full analysis with markets, patents, and monopoly power is informative, but unfortunately it lies beyond the scope of an intermediate macro text.¹¹ Instead, we make a simplifying assumption and allocate labor through a rule of thumb, the same way Solow allocated output to consumption and investment. In particular,

¹⁰The result that the market allocation provides too little incentive for research depends on the exact model and the exact institutions for allocating resources that one introduces into the model. Nevertheless, most empirical work that has looked at this question has concluded that advanced economies like the United States probably underinvest in research. For a survey of recent research, see Vania Sena, "The Return of the Prince of Denmark: A Survey on Recent Developments in the Economics of Innovation" *The Economic Journal*, June 2004, Volume 114, pp. F312-F332.

¹¹The interested reader might find it helpful to look at Chapter 5 of my textbook on growth: Charles I. Jones, *Introduction to Economic Growth* (W. W. Norton, 2002), which works through this analysis.

Table 6.1: The Romer Model: 4 Equations and 4 Unknowns

Unknowns/Endogenous Variables: Y_t, A_t, L_{yt}, L_{at}	
The output production function	$Y_t = A_t L_{yt}$
The idea production function	$\Delta A_t = \bar{z} A_t L_{at}$
The resource constraint	$L_{yt} + L_{at} = \bar{N}$
Allocation of labor	$L_{at} = \bar{\ell} \bar{N}$
Parameters: $\bar{z}, \bar{N}, \bar{\ell}, \bar{A}_0$	

we assume the constant fraction $\bar{\ell}$ of the population works in research, leaving $1 - \bar{\ell}$ to work in producing output. For example, we might set $\bar{\ell} = .05$, so that 5% of the population works to produce new ideas, while 95% of the population works to produce consumption goods. Our fourth equation is therefore

$$L_{at} = \bar{\ell} \bar{N}. \quad (6.5)$$

This completes our description of the Romer model. It is summarized in Table 6.1.

6.3.1 Solving the Romer Model

To solve this model, we need to express our four endogenous variables as functions of the parameters of the model and time. Fortunately, our model is simple enough that this can be done easily. First, notice that $L_{at} = \bar{\ell} \bar{N}$ and $L_{yt} = (1 - \bar{\ell}) \bar{N}$. Those are the solutions for two of our endogenous variables.

Next, using the production function in equation (6.2), per capita output can be written as

$$y_t \equiv \frac{Y_t}{\bar{N}} = A_t (1 - \bar{\ell}). \quad (6.6)$$

This equation says that per capita output is proportional to A_t . That is, output *per person* depends on the *total* stock of knowledge. So a new idea that increases A_t will raise the income of each person in the economy. This feature of the model reflects the

nonrivalry of ideas: each idea can be used by each person in the economy. Contrast this with the Solow model, where per capita income depended on capital *per person* rather than on the total capital stock.

Finally, to complete our solution for the model, we need to solve for the stock of knowledge A_t at each point in time. Dividing the production function for ideas in equation (6.3) by A_t yields

$$\frac{\Delta A_t}{A_t} = \bar{z}L_{at} = \bar{z}\bar{\ell}\bar{N}. \quad (6.7)$$

This equation says that the growth rate of the stock of knowledge is constant over time. It is proportional to the number of researchers in the economy, L_{at} , which in turn is proportional to the population of the economy, \bar{N} . It is useful to define $\bar{g} \equiv \bar{z}\bar{\ell}\bar{N}$ as this particular combination of parameters so that we do not have to keep writing out each of these terms.

Since the growth rate of knowledge is constant over time, even starting from time 0, the stock of knowledge is therefore given by

$$A_t = \bar{A}_0(1 + \bar{g})^t, \quad (6.8)$$

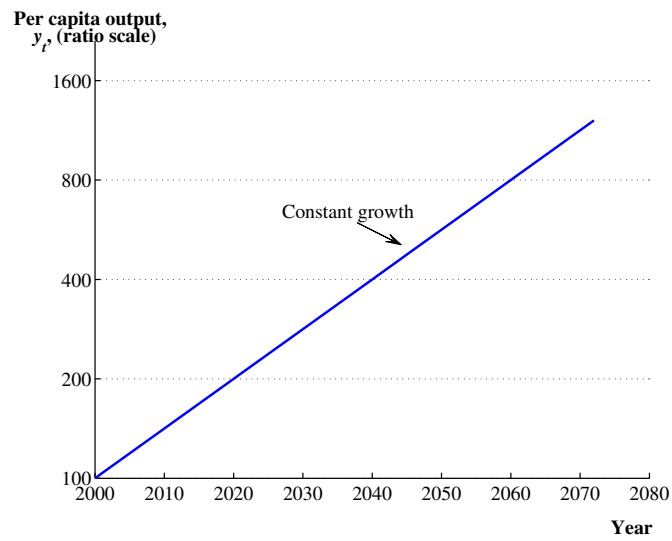
where the growth rate \bar{g} is defined above. If you have trouble understanding where this equation comes from, notice that it is just an application of the *constant growth rule* from Chapter ???. That is, since we know A_t grows at a constant rate, the level of A_t is equal to its initial value multiplied by $(1 + \bar{g})^t$, where \bar{g} is the growth rate.

This last equation, together with equation (6.6) for per capita output, completes the solution to the Romer model. In particular, combining these two equations, we have

$$\boxed{y_t = \bar{A}_0(1 - \bar{\ell})(1 + \bar{g})^t}, \quad (6.9)$$

where $\bar{g} \equiv \bar{z}\bar{\ell}\bar{N}$. The level of per capita output is now written entirely as a function of the parameters of the model. Figure 6.2 uses this solution to plot output per capita over time for the Romer model. It shows up as a straight line on a ratio scale, since output per person grows at a constant rate at all times.

Figure 6.2: Per Capita Output in the Romer Model



Note: The vertical axis of the figure has a ratio scale, so a straight line represents constant growth. Per capita output in the year 2000 is normalized to take a value of 100.

6.3.2 Why is there Growth in the Romer Model?

Now that we have solved the Romer model, let's think about what the solution means. First and foremost, we have a theory of sustained growth in per capita income. This holy grail that we have been searching for over the last several chapters has finally been obtained.

This is the main result of this chapter, and you should pause to appreciate its elegance and importance. The nonrivalry of ideas means that per capita income depends on the *total* stock of ideas. Researchers produce new ideas, and the sustained production of new ideas leads to the sustained growth of income over time. Romer's division of goods into objects and ideas leads to a theory of long-run growth in a way that Solow's division into capital and labor did not.

Why is this the case? Why does the Romer model generate sustained growth endogenously while the Solow model could not. Recall that in the Solow model, the accumulation of capital runs into diminishing returns: each new addition to the capital stock increases output — and therefore investment — by less and less. Eventually, these additions are just enough to offset the depreciation of capital. Since new investment and depreciation offset, capital stops growing, and so does income.

In the Romer model, consider the accumulation equation for new ideas:

$$\Delta A_t = \bar{z} A_t L_{at}.$$

There are no diminishing returns to the existing stock of ideas in the production of new ideas — the exponent on A in this production function is equal to one. As we accumulate more knowledge, the return to knowledge does not fall. Old ideas continue to help us produce new ideas in a virtuous circle that sustains economic growth.

Why does capital run into diminishing returns in the Solow model but ideas do not in the Romer model? The answer is nonrivalry. Capital is an object, and the standard replication argument tells us there are constant returns to all objects taken together: labor and capital. Therefore there are diminishing returns to capital by itself.

In contrast, the nonrivalry of ideas means that there are increasing returns to ideas and objects together. This places no restriction on the returns to ideas, allowing for the

possibility explored here that the returns to accumulating ideas do not fall. Because the accumulation of ideas does not run into diminishing returns, growth can be sustained.

———— BOX: A Model of World Knowledge ————

How should we think about applying this model to the world? This is an important question that needs to be considered carefully. For example, suppose we thought of applying this model to each country individually. What would it imply about the difference between Luxembourg and the United States? Luxembourg is a country with fewer than half a million people, while the United States has a population that is more than 600 times larger. According to statistics from the National Science Foundation, there are more researchers in the United States than Luxembourg has people! Since the growth rate of per capita income in the model is tied to the number of researchers, the model would seem to predict that the United States should grow at a rate that is several hundred times larger than the growth rate of Luxembourg. This is obviously not true. Between 1960 and 2000, the growth rate of per capita GDP in the United States averaged 2.5 percent per year, while growth in Luxembourg was nearly a full percentage point faster, at 3.3 percent.

A moment's thought, though, reveals the error we've made in this application. It is not the case that the economy of Luxembourg grows only because of ideas invented in Luxembourg, or even that the United States grows only because of ideas invented by U.S. researchers. Instead, virtually all countries in the world benefit from ideas created throughout the world. International trade, multinational corporations, licensing agreements, international patent filings, the migration of students and workers, and the open flow of information ensure that an idea created in one place can impact economies worldwide.

It is more accurate to think of the Romer model as a model of the world's stock of ideas. Through the spread of these ideas, growth in the world's stock of knowledge drives the underlying trend rate of growth in every country in the world. Why, then, do countries grow at different rates? We will see more about this in the next section, but a hint at the answer is this: we will invoke the transition dynamics of the Solow model.

———— End of Box ————

6.3.3 Experiments in the Romer Model

What happens if we set up a Romer economy, watch it evolve for awhile, and then change one of the underlying parameter values? Before answering this question, let's make one other related remark about how the Romer economy evolves. In particular, notice that, unlike the Solow model, the Romer model does not exhibit transition dynamics. In the Solow model, the economy would start out by growing (e.g. if we started below the steady state), and then the growth rate would gradually decline as the economy approached its steady state.

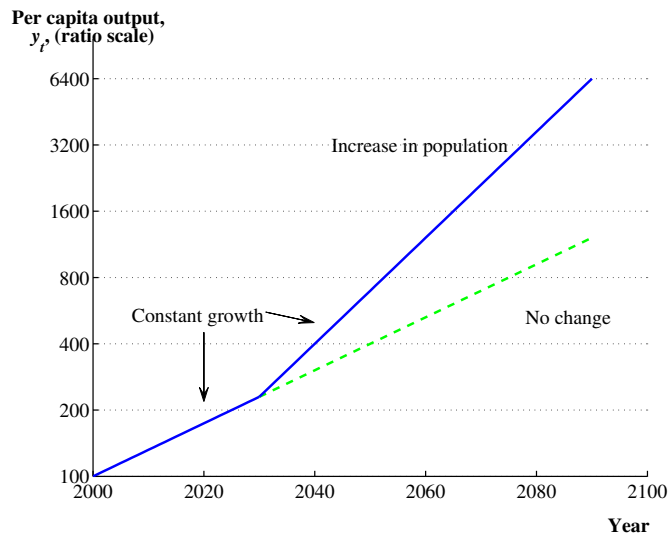
In the Romer model, in contrast, the growth rate is constant and equal to $\bar{g} \equiv \bar{v}\bar{\ell}\bar{N}$ at all points in time — take a look back at equation (6.9). The growth rate never rises or falls, and in some sense the economy could be said to be in its steady state from the very start. Of course, because the model feature sustained growth, it is a little odd to call this a steady state. For this reason, economists sometimes refer to the economy as being on a “balanced growth path.” A *balanced growth path* is defined as a situation in which the growth rates of all endogenous variables are constant. The Romer economy is on its balanced growth path at all times, and growth rates do not rise or fall over time (under our maintained assumption that the parameter values are unchanged).

Now let's see what happens when we change a parameter value. There are four parameters in the model: \bar{N} , $\bar{\ell}$, \bar{z} , and \bar{A}_0 . We will discuss the first two of these and leave the others as exercises at the end of the chapter.

Experiment #1: Changing the Population, \bar{N}

For our first experiment, let's consider an exogenous increase in the population, \bar{N} , holding all other parameter values constant. In looking at the solution to the Romer model in equation (6.9), \bar{N} shows up in one place, the growth rate of knowledge. Because the research share is held constant, a larger population means there are more researchers. More researchers produce more ideas, and this leads to faster growth: a Romer economy with more researchers actually grows faster over time. Figure 6.3 shows the effect of this experiment on the time path for per capita output.

Figure 6.3: Per Capita Output after an Increase in \bar{N}



Note: The figure considers a one-time, permanent increase in \bar{N} that occurs in the year 2030. Notice that the vertical axis of the figure has a ratio scale, so a straight line represents constant growth. Per capita output in the year 2000 is normalized to take a value of 100.

We have already discussed the fact that one does not want to apply the Romer model on a country by country basis. Instead, it is better to view it as a model of the world as a whole. With this perspective, the model may help us to understand economic growth over the long course of history. Recall from Figure ?? in Chapter ??, that over the last several thousand years, growth rates have been rising. Michael Kremer of Harvard University has suggested that this could be the result of virtuous circle between ideas and population. People create new ideas, and new ideas make it possible for finite resources to support a larger population. The larger population in turn creates even more ideas, leading to the virtuous circle.¹²

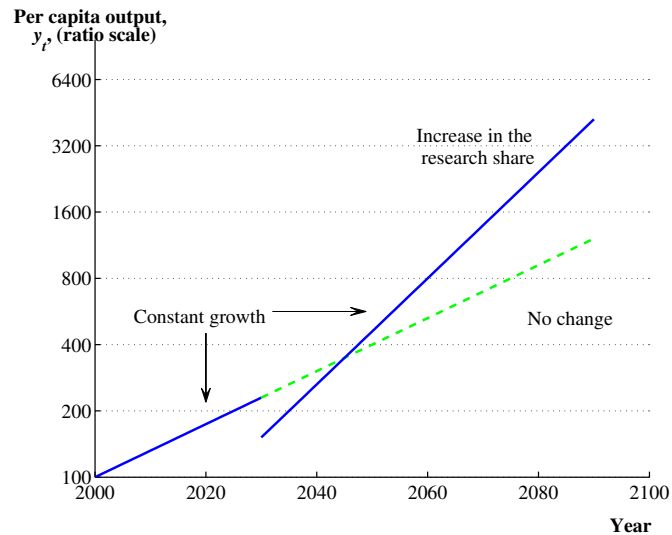
That said, the careful reader may challenge the validity of this prediction for the last century or so. Both the world population and the overall number of researchers in the world have increased dramatically during the 20th century. According to the Romer model, the growth rate of per capita GDP should therefore have risen sharply as well. However, this is clearly not the case. For example, recall that one of our key stylized facts is that U.S. growth rates have been relatively stable over the last hundred years. Fortunately, extensions of the Romer model can render it consistent with this evidence, as discussed in more detail below.

Experiment #2: Changing the Research Share, $\bar{\ell}$

For our second experiment, let's suppose the fraction of labor working in the ideas sector, $\bar{\ell}$, increases. Look back at our solution in equation (6.9) to see what happens. There are two effects. First, there are now more researchers, so more new ideas are produced each year. This leads the growth rate of knowledge, $\bar{g} \equiv \bar{z}\bar{\ell}\bar{N}$, to rise. From equation (6.9), this also causes the growth rate of per capita income to rise, just as in our previous experiment.

The second effect of the increase in the research share of the population is less obvious, but it can also be seen in equation (6.9). In particular, if more people are working to produce ideas, fewer are available to produce the consumption good. This means that the *level* of output per capita declines. Increasing the research share thus

¹²Michael Kremer, "Population Growth and Technological Change: One Million B.C. to 1990," *Quarterly Journal of Economics*, Volume 108, August 1993, pp. 681-716.

Figure 6.4: Per Capita Output after an Increase in $\bar{\ell}$ 

Note: The figure considers a one-time, permanent increase in $\bar{\ell}$ that occurs in the year 2030. Two things happen. First, the growth rate is higher: more researchers produce more ideas, leading to faster growth. Second, the initial level of per capita output declines: there are fewer workers in the consumption goods sector, so production per capita must fall initially. Notice that the vertical axis of the figure has a ratio scale, so a straight line represents constant growth. Per capita output in the year 2000 is normalized to take a value of 100.

involves a tradeoff: current consumption declines, but the growth rate of consumption is higher, so future consumption is higher as well.¹³ The results of this experiment are shown graphically in Figure 6.4.

6.3.4 Growth Effects versus Level Effects

Our discussion of the importance of increasing returns in generating sustained growth finessed one important issue. The careful reader may have noticed that in the two production functions of the Romer model, not only are there increasing returns to ideas and objects, but the degree of increasing returns is especially strong. (Take a look back

¹³With some additional mathematics, one can think about the optimal value for the research share, but for now we will simply note that the optimal value is at some midpoint: an economy needs researchers to produce ideas in order to raise future income, but it also needs workers to produce output today in order to facilitate current consumption.

at Table 6.1 to review these production functions.)

The standard replication argument tells us that there should be constant returns to objects (labor) in these production functions, and therefore increasing returns to labor and ideas together. This means the exponent on labor in these production functions should be unity, but it says nothing about what the exponent on ideas should be, other than positive.

In particular, an interesting question to ask is this: what would happen if the exponent on ideas were equal to some number less than 1? For example, what if $\Delta A_t = \bar{z} A_t^{1/2} L_{at}$? Notice that there would still be increasing returns overall (the exponents add to more than one), but there would also be diminishing returns to ideas alone.

This is an important question, and the answer comes in two parts. First, the ability of the Romer model to generate sustained growth in per capita income is not sensitive to the degree of increasing returns. The general point we have been making in this chapter — that the nonrivalry of ideas leads to increasing returns, and this delivers a theory of sustained growth — is robust to changing the “strength” of increasing returns. The Romer framework provides a robust theory of long-run growth.

Our second point, though, is that other predictions of the Romer model are sensitive to the degree of increasing returns. For example, in the two experiments we just considered, changes in parameters led to permanent increases in the rate of growth of per capita income.

These “growth effect” results can be eliminated in models when the degree of increasing returns is not strong. If the exponent on the stock of ideas in the idea production function is less than one, increases in ideas run into diminishing returns. In such specifications, an increase in the research share or the size of the population increases the growth rate in the short run, but in the long run the growth rate returns to its original value. This is very much like the transition dynamics of the Solow model, and it occurs for much the same reason. More researchers produce more ideas, and this raises the long-run *level* of per capita income. The long-run growth rate is positive in this alternative model, but it is unchanged by a one-time increase in the number of

researchers.¹⁴

6.3.5 Summary

The Romer model divides economic goods into two categories: objects and ideas. Objects are the raw materials available in an economy, and ideas are ways of using these raw materials to generate utility. The Solow approach studies a model based solely on objects — raw materials — and finds that it cannot provide a theory of sustained growth in per capita income. The Romer approach shows that the discovery of new ideas — better ways to use the raw materials that are available to us — can provide a theory of long-run growth.

The key reason why the idea approach succeeds where the object approach fails is that ideas are nonrivalrous. That is, the same set of ideas can be used by one production line or two production lines or a hundred production lines. By the standard replication argument, there are constant returns to scale to objects in production. But this means there are increasing returns to object and ideas taken together.

Because of nonrivalry and increasing returns, each new idea has the potential to increase the income or utility of every person in an economy. It is not “ideas per capita” that matter for an individual’s income and well-being, but rather the total stock of ideas. Sustained growth in the stock of knowledge, then, is the key to sustained growth in per capita income.

6.4 Combining Romer and Solow

In Chapter ??, we used the Solow model to understand many important issues related to growth, including why countries grow at different rates for decades at a time and why some countries are richer than others. That model, however, left unanswered the critical question of how growth can be sustained in the long run. The Romer model answers this latter question.

¹⁴Many of the general results in this chapter go through in this alternative version of the Romer model. As this section hints, however, there are some important differences as well. The interested reader may consult Chapter 5 of Charles I. Jones *Introduction to Economic Growth* (W. W. Norton, 2002) and Charles I. Jones “R&D-Based Models of Economic Growth” *Journal of Political Economy*, August 1995, Volume 103, pp. 759-784.

In presenting the Romer model, we made the simplifying assumption that there was no capital in the economy. This is helpful for seeing the insights of the Romer model. However, because the Solow model also helps us answer many questions about economic growth, it is important to understand how to combine the Solow and Romer frameworks.

An optional appendix at the end of this chapter details how the insights of Solow and Romer can be combined in a single model of economic growth. Intuitively, all of the results we have learned from both models continue to hold in the combined model. However, the appendix uses algebra more intensively than elsewhere in the book, so it is an optional section. With this in mind, we summarize here the key results that are obtained with the combined model. This summary should be informative even for readers who will not be working through the appendix.

In the combined Solow-Romer Model, long-run growth is sustained because of the nonrivalry of ideas. The invention of new ideas is once again the key to long-run growth.

The most important element that enters the combined model from the Solow side is the principle of transition dynamics. Because of the diminishing returns to capital accumulation, transition dynamics are an important feature of the combined model. If an economy starts out below its balanced growth path, it will grow rapidly in order to catch up to this path. Conversely, if an economy begins above its balanced growth path, it will grow slowly for a period of time.

The Romer model helps us to understand the overall trend in incomes around the world and why growth is possible. The transition dynamics of the Solow model help us to understand why Japan and Korea have grown faster than the United States for the last half century. In the long run, all countries grow at the same rate. But because of transition dynamics, actual growth rates can differ across countries for long periods of time.

6.5 Growth Accounting

One of the many useful ways in which growth models like the combined Solow-Romer model have been applied is to understand the sources of growth in a particular economy and how they may have changed over time. The particular approach outlined in this section is called *growth accounting*. Robert Solow was one of the early economists to apply this technique to the United States.¹⁵

To see how growth accounting works, consider a production function that includes both capital and ideas:

$$Y_t = A_t K_t^{1/3} L_{yt}^{2/3}. \quad (6.10)$$

The term A_t can be thought of as the stock of ideas, as in the Romer model, or more generally as the level of total factor productivity.

We now apply the rules for computing growth rates that we developed back in Chapter ?? to the production function for output. In fact, one of the examples we worked out in that chapter is exactly the problem we have before us now. If you do not understand the derivation in the next paragraph, take a look back at the example in Section ?? of Chapter ?? and the steps are laid out there in detail.

We use two of the rules for computing growth rates: the growth rate of a product is the sum of the growth rates of the terms, and the growth rate of a variable raised to some power is that power times the growth rate of the variable. Applied to the production function for output in equation (6.10), we have

$$g_{Yt} = g_{At} + \frac{1}{3}g_{Kt} + \frac{2}{3}g_{L_{yt}}, \quad (6.11)$$

where $g_{Yt} \equiv \Delta Y_t / Y_t$, and the other growth rates are defined in a similar way.

Equation (6.11) is really just the growth rate version of the production function. It says that the growth rate of output is the sum of three terms: the growth rate of total factor productivity, the growth contribution from capital, and the growth contribution from workers. Notice that the growth contributions of capital and workers get weighted

¹⁵See Robert M. Solow, “Technological Change and the Aggregate Production Function” *Review of Economics and Statistics*, 1957, Volume 39, pp. 312–320. Other important early contributors to growth accounting include Edward Denison and Dale Jorgenson. The Bureau of Labor Statistics now conducts these growth accounting exercises at regular intervals for the United States.

by their exponents in the production function, reflecting the diminishing returns to each of these inputs.

Suppose, as is true in practice, that the number of hours worked by the labor force can change over time. For example, when the economy is booming, people may work more hours per week than when the economy is in a recession. Let L denote the aggregate number of hours worked by everyone in the economy, and let g_L denote the growth rate of hours worked. Now, subtract $g_L t$ from both sides of equation (6.11) above to get

$$\underbrace{g_{Yt} - g_{Lt}}_{\text{Growth of Y/L}} = \underbrace{\frac{1}{3}(g_{Kt} - g_{Lt})}_{\text{Contribution from K/L}} + \underbrace{\frac{2}{3}(g_{Ly t} - g_{Lt})}_{\text{Labor composition}} + \underbrace{g_{At}}_{\text{TFP Growth}} . \quad (6.12)$$

In this equation, notice that on the right-side we've used the fact that $g_L = 1/3 \times g_L + 2/3 \times g_L$ in the subtraction. Also, we've moved the TFP growth term to the end.

This last equation tells us that the growth rate of output per hour, Y/L , over a time period can be viewed as the sum of three terms. The first term is the contribution from the growth of capital per hour worked by the labor force. As capital per hour rises, output per hour rises as well, but this effect is reduced according to the degree of diminishing returns to capital, $1/3$. The second term is called a “labor composition” term. Here, we've written it as the growth rate of workers less the growth rate of total hours, but in actual applications of growth accounting, this term can also include increases in the educational attainment of the labor force or changes in the age distribution of the workforce. This is why we use the term “labor composition” rather than hours worked per worker. Finally, the last term in the equation is the growth rate of total factor productivity. Faster productivity growth also raises the growth rate of output per hour.

By measuring the growth rates of output per hour, capital per hour, and the changing composition of the labor force, we can use this equation to account for the sources of growth in a given country. In practice, all of the terms other than TFP growth can be measured empirically, so this equation is used to calculate TFP growth. For this reason, TFP growth is sometimes called “the residual.” You may recall that we discussed

Table 6.2: Growth Accounting for the United States

	1948–02	1948–73	1973–95	1995–02
Output per hour, Y/L	2.5	3.3	1.5	3.0
Contribution of K/L	0.9	0.9	0.7	1.3
Contribution of Labor composition	0.2	0.2	0.2	0.4
Contribution of TFP, A	1.4	2.1	0.6	1.2

Note: The table shows the average annual growth rate (in percent) for different variables. Source: Bureau of Labor Statistics, *Multifactor Productivity Trends, 2002*, February 1, 2005.

a similar point when we used the production function to account for differences in levels of per capita GDP back in Chapter ???. Accounting for growth involves the same reasoning: we can observe everything *other* than TFP, so we use our equation as a way to *measure* the unobserved TFP growth.

Table 6.2 shows this decomposition for the United States, first for the entire period of 1948 to 2002, and then for particular subperiods. Output per hour in the United States grew at an average annual rate of 2.5 percent between 1948 and 2002. Of this 2.5 percent, 0.9 percentage points were due to an increase in the capital-labor ratio, K/L , and an additional 0.2 percentage points came from changes in the composition of the labor force, including increased educational attainment. This means that the residual, TFP growth, accounted for the majority of growth, coming in at 1.4 percentage points.

Next, notice that these numbers have changed over time in interesting ways. First, the period 1948 to 1973 featured the fastest growth in output per hour in the table, at 3.3 percent per year. The years 1973 to 1995 saw output per hour grow less than half as fast, at 1.5 percent per year. What accounted for the slowdown in growth? As shown in the table, nearly the entire slowdown is explained by a decline in total factor productivity growth, from a rapid rate of 2.1 percent per year before 1973 to an anemic rate of 0.6 percent per year after. Economists refer to this episode as the *productivity slowdown*. It has been studied in great detail in an effort to understand exactly what caused productivity growth to decline so dramatically. The list of potential explanations is long and includes the oil price shocks of the 1970s, a decline in the fraction of GDP spent on research and development, and a change in the sectoral composition of the

economy toward services and away from manufacturing. Though each of these factors seems to have contributed to the slowdown, the exact cause(s) remain elusive.¹⁶

Just as remarkable as the slowdown, however, is the dramatic resurgence of growth that has occurred since 1995. Between 1995 and 2002, output per hour grew at an annual rate of 3.0 percent, nearly as rapidly as before the productivity slowdown. This era, of course, was marked by the rise of the world wide web and the dot.com boom in the stock market, followed by the sharp decline in stock prices in 2001. Overall, some commentators have labeled this era the *new economy* to mark the resurgence of growth.

What accounted for the resumption of relatively rapid growth in the United States? In the accounting exercise, the increase in growth is accounted for in roughly equal parts by capital accumulation and TFP. The contribution of capital rose from 0.7 to 1.3 percentage points, while total factor productivity growth increased from 0.6 to 1.2 percentage points. Economists studying this productivity boom generally conclude that at least half of this increase in growth can be explained by purchases of computers and information technology and by rapid productivity growth in the sectors that produce information technology products. That is, there is a link between the new economy and information technology.¹⁷

6.6 Concluding our Study of Long-Run Growth

The key to sustained growth in per capita income is the discovery of new ideas. Because ideas are nonrivalrous, it is not ideas per capita that matter for individual welfare, but rather the total stock of ideas. Increases in the stock of knowledge lead to sustained economic growth for countries that have access to that knowledge. This is the lesson of the Romer model.

Combining the insights of the Solow model and the Romer model leads to our full theory of long-run economic performance. Each country can be thought of as a Solow economy that sits on top of the overall trend in world knowledge that is generated by a

¹⁶See the Fall 1998 issue of the *Journal of Economic Perspectives* for a more detailed discussion of the productivity slowdown.

¹⁷For a discussion of the causes and consequences of the recent boom in productivity growth, see William Nordhaus, "The Sources of the Productivity Rebound and the Manufacturing Employment Puzzle" NBER Working Paper 11354, May 2005, and Robert J. Gordon, "Five Puzzles in the Behavior of Productivity, Investment, and Innovation" NBER Working Paper 10660, August 2004.

Romer model. Growth in the stock of knowledge accounts for the overall trend in per capita income over time. Transition dynamics associated with the Solow model allow us to understand differences in growth rates across countries that persist for several decades.

The United States and Korea have both experienced sustained growth in per capita income, driven in large part by the increase in the world's stock of knowledge. Korea has grown faster than the United States during the last four decades because structural changes in the economy have shifted its balanced growth path sharply upward. Whereas in 1950 the steady-state ratio of per capita income in Korea to per capita income in the United States may have been 10 or 15 percent, the steady-state ratio today is probably some number like 80 percent (the exact number depends on Korea's investment rate and productivity level, among other things). The Korean economy has grown rapidly during the last several decades as it makes the transition from its initial low steady-state income ratio to its eventual high ratio.

Nigeria, on the other hand, is almost the opposite story. In the 1950s and early 1960s, Nigeria had a per capita income that was just under 10 percent of the U.S. level. Since then, however, Nigeria's economy has steadily deteriorated so that by 2000 per capita income was only slightly more than 2 percent of the U.S. level and roughly at the same level that it was in 1950. A half century of lost growth is a tremendous lost opportunity, as this is period when U.S. living standards more than tripled while those in Korea rose by a factor of 12. The Solow model suggests that this tragedy has its roots in a decline in the investment rate in physical capital and a decline in productivity, at least relative to the rest of the world. Part of this decline in productivity may come from a change in the degree to which Nigerians can access the world's stock of ideas, a point suggested by the Romer model.

Of course exactly what caused these changes in Korea or Nigeria is a critical subject that our growth model does not speak to. Recent research in the economics literature has focused on the important role played by *institutions*. For example, the extent to which property rights are protected and the extent to which contractual agreements between willing parties are enforced by the law appear to play an important role in determining investment and productivity. In the absence of these institutions, firms may

be unwilling to make investments in an economy and the transfer of knowledge that often seems to come with trade and foreign investment may be hindered. At the same time, improvements in these institutions may help explain the increase in investment and productivity that our Solow-Romer model suggests is associated with rapid growth. The role of institutions in explaining differences in incomes was discussed in more detail at the end of Chapter ??.¹⁸

This is where our study of economic growth leaves off. The Solow and Romer frameworks provide a sound foundation for understanding why some countries are so much richer than others and why economies grow over time. This is not to say that these models provide the final answers. Questions of why countries have different investment rates and total factor productivity levels remain unanswered — and remain at the frontier of economic research. But there is more macroeconomics to cover and our time is short.

The next two chapters maintain the focus on the long run, but turn to other aspects of the economy. In particular, Chapter ?? examines the labor market, while Chapter ?? studies the determinants of inflation and nominal interest rates in the long run.

6.7 A Postscript on Solow and Romer

For the purpose of explaining the main contributions of the Solow model and the Romer model, we have simplified both models in ways that do not do justice to the richness of either of the original papers. First, the basic production model presented in Chapter ?? is itself a contribution of the original Solow paper. Second, as we have discussed, Solow endogenized capital accumulation and found that capital could not provide the engine of economic growth. But he went further in postulating that exogenous improvements in technology — “exogenous technological progress” — could explain growth. That is, Solow included an equation like $\Delta A_t/A_t = .02$, allowing productivity to grow at a constant, exogenous rate of 2% per year. The limitation of this approach, however, is that the growth was simply assumed rather than explained within the model. Solow intuited that it must be related to technological improvements, but he took these to be

¹⁸A section at the end of this chapter contains additional readings for the student wishing to pursue this topic further.

exogenous, like rain falling from the sky.

The original Romer paper was also significantly richer than our presentation allows. As a minor example, Romer included capital accumulation in his original model. More importantly, Romer solved a significant puzzle related to increasing returns. That is, if increasing returns is the key to growth, why wouldn't the economy come to be dominated by a single, very large firm? After all, such a firm would take the best advantage of increasing returns. Romer's answer was to incorporate the modern theory of monopolistic competition into his idea model. The economy contains many monopolists, each producing slightly different goods. They compete with each other to sell us different varieties of music, books, computers, and airplanes. Their size, as Adam Smith said, is limited by the extent of the market as well as by competition with each other.

6.8 Additional Resources

There are a number of interesting and useful readings related to economic growth. Some are collected here and organized loosely by topic.

Ideas: Interesting readings related to ideas and growth include the following:

- Paul Romer's web page contains links to a number of short articles on economic growth and ideas: <http://www.stanford.edu/~promer>. For example, the quote that began this chapter is taken from an encyclopedia article on economic growth that he wrote. It can be found at <http://www.stanford.edu/~promer/Econgro.htm>.
- Warsh, David. *Knowledge and the Wealth Of Nations: A Story of Economic Discovery*. New York: W.W. Norton, 2006.
- Mokyr, Joel. *The Gifts of Athena: Historical Origins of the Knowledge Economy*. Princeton, N.J.: Princeton University Press, 2002.
- Simon, Julian L. *The Ultimate Resource 2*. Princeton, N.J.: Princeton University Press, 1998.

Institutions and economic growth: We only had a chance in this chapter to mention briefly this very interesting research that is currently at the frontier of the study of

economic growth. The interested reader may pursue this topic further by taking a look at

- Mancur Olson, “Distinguished Lecture on Economics in Government: Big Bills Left on the Sidewalk: Why Some Nations Are Rich, and Others Poor,” *Journal of Economic Perspectives*, Spring 1996, Vol. 10(2), pages 3–24.
- Chapter 7 of Charles I. Jones, *Introduction to Economic Growth*. New York: W.W. Norton, 2001.
- Douglass C. North and Robert P. Thomas, *Institutions, Institutional Change, and Economic Performance*. Cambridge University Press, 1990.
- Stephen L. Parente and Edward C. Prescott, *Barriers to Riches*. Cambridge, MA: MIT Press, 2000.
- Daron Acemoglu and James A. Robinson, *Economic Origins of Dictatorship and Democracy*. Cambridge University Press, 2005.

Other references. Finally, there are a number of other fascinating books related to growth that are of broad interest. Some examples are

- William Easterly, *The Elusive Quest for Growth: Economists’ Adventures and Misadventures in the Tropics*. Cambridge, MA: MIT Press, 2001.
- Elhanan Helpman, *The Mystery of Economic Growth* Belknap Press, 2004.
- Jared Diamond, *Guns, Germs, and Steel*. New York: W.W. Norton, 1997.

6.9 Summary

1. Whereas Solow divides the world into capital and labor, Romer divides the world into ideas and objects. This distinction proves to be essential for understanding the engine of growth.
2. The Idea Diagram is a useful organizing device for this chapter. *Ideas* are instructions for combining objects in ways that generate utility. Existing ideas are

nonrivalrous; they are not scarce in the same way that objects are. Instead, ideas can be used by an arbitrary number of people simultaneously without anyone's use being degraded.

3. This nonrivalry implies that the economy is characterized by increasing returns to ideas and objects taken together. There are fixed costs associated with inventing ideas that are a reflection of this increasing returns.
4. Increasing returns implies that Adam Smith's invisible hand may not lead to the best of all possible worlds. Prices must be above marginal cost in some places in order for firms to recoup the fixed cost of inventing ideas. If a pharmaceutical company were to charge marginal cost for its drugs, it would never be able to cover the large cost of inventing drugs in the first place.
5. Growth eventually ceases in the Solow model because capital runs into diminishing returns. Because of the nonrivalry of ideas, ideas need not run into diminishing returns, and this allows growth to be sustained.
6. Combining the insights from Solow and Romer leads to a rich theory of economic growth. The growth of world knowledge explains the underlying upward trend in incomes. Countries can grow faster or slower than this world trend because of the principle of transition dynamics.

6.10 Key Concepts

The Idea Diagram, ideas, objects, nonrivalry, increasing returns, problems with perfect competition, fixed costs, the Romer model, balanced growth path, growth effects, level effects, the principle of transition dynamics, growth accounting, the productivity slowdown, the new economy

6.11 Review Questions

1. How are ideas different from objects? What are some examples of each?

2. What is nonrivalry, and how does it lead to increasing returns? What role does the standard replication argument play? Is national defense rivalrous or nonrivalrous?
3. Suppose a friend of yours decides to write a novel. Explain how ideas and objects are involved in the production of novels. Where do nonrivalry and increasing returns play a role? What happens if the novel is sold at marginal cost?
4. Explain how nonrivalry leads to increasing returns in the two key production functions of the Romer model.
5. The growth rate of output in the Romer model is $\bar{v}\bar{\ell}\bar{N}$. Why do each of these parameters enter the solution for the growth rate?
6. Why is growth accounting useful?

6.12 Exercises

1. *Nonrivalry*. Explain whether the following goods are rivalrous or nonrivalrous: (a) Beethoven's Fifth Symphony, (b) a portable music player, (c) a famous Monet impressionist painting, (d) the method of public key cryptography, (e) fish in the ocean.
2. *Increasing returns and imperfect competition*. Suppose a new piece of computer software — say a word processor with perfect speech recognition — can be created for a one-time cost of \$100 million. Suppose that once it is created, copies of the software can be distributed at a cost of \$1 each (this is probably too high, but it's a nice round number for the problem).
 - (a) If Y denotes the number of copies of the computer program produced and X denotes the amount spent on production, what is the production function, i.e. the relation between Y and X ?
 - (b) Make a graph of this production function. Does it exhibit increasing returns? Why or why not?

- (c) Suppose the firm charges a price equal to marginal cost and sells a million copies of the software. What are its profits?
- (d) Suppose the firm charges a price of \$20 for each copy of the computer program. How many copies does the firm have to sell in order to break even? What if the price is \$100 per copy?
- (e) Why does the scale of the market matter?

3. *Calculating growth rates.* What is the growth rate of per capita output in Figure 6.2? Also, calculate the growth rate of per capita output before and after the changes in the parameter values in Figure 6.3 and Figure 6.4.

4. *An increase in research productivity.* Consider the basic Romer model in this chapter. Suppose the economy is on a balanced growth path and then, in the year 2030, research productivity \bar{z} rises immediately and permanently to the new level \bar{z}' .

- (a) Solve for the new growth rate of knowledge and y_t .
- (b) Make a graph of y_t over time using a ratio scale.
- (c) Why might research productivity increase in an economy?

5. *An increase in the initial stock of knowledge.* Suppose we have two economies — let's call them Earth and Mars — that are identical, except that one begins with a stock of ideas that is twice as large as the other: $\bar{A}_0^{\text{earth}} = 2 \times \bar{A}_0^{\text{mars}}$. The two economies are so far apart that they do not share ideas, and each evolves as a separate Romer economy. On a single graph (with a ratio scale), plot the behavior of per capita income on Earth and Mars over time. What is the effect of starting out with more knowledge?

worked exercise

6. *Putting some numbers into the Romer model.* Consider the basic Romer model studied in this chapter. Suppose the parameters of the model take the following values: $\bar{A}_0 = 100$, $\bar{\ell} = .10$, $\bar{z} = 1/500$, and $\bar{N} = 100$.

- (a) What is the growth rate of per capita output in this economy?

- (b) What is the initial level of per capita output? What is the level of per capita output after 100 years?
- (c) Suppose the research share were to double. How would you answer to the two previous parts of this problem change?

7. *More numbers in the Romer model.* Consider the basic Romer model studied in this chapter. Suppose the parameters of the model take the following values: $\bar{A}_0 = 100$, $\bar{\ell} = .06$, $\bar{z} = 1/3000$, and $\bar{N} = 1000$.

- (a) What is the growth rate of per capita output in this economy?
- (b) What is the initial level of per capita output? What is the level of per capita output after 100 years?
- (c) Now consider the following changes, one at a time: a doubling of the initial stock of knowledge \bar{A}_0 , a doubling of the research share, a doubling of research productivity \bar{z} , and a doubling of the population. How would your answer to the two previous parts of this problem change in each of these cases?
- (d) If you could advocate one of the changes considered in part (c), which would you choose? Write a paragraph arguing for your choice.

8. *A variation on the Romer model.* Consider the following variation on the Romer model:

$$Y_t = A_t^{1/2} L_{yt}$$

$$\Delta A_t = \bar{z} A_t L_{at}$$

$$L_{yt} + L_{at} = \bar{N}$$

$$L_{at} = \bar{\ell} \bar{N}.$$

There is only a single difference: we've changed the exponent on A_t in the production of the output good so that there is now a diminishing marginal product to ideas in that sector.

- (a) Provide an economic interpretation for each equation.

- (b) What is the growth rate of knowledge in this economy?
 - (c) What is the growth rate of per capita output in this economy?
 - (d) Solve for the level of per capita output at each point in time.
9. *Growth accounting.* Consider the following (made-up) statistics for some economies. Assume the exponent on capital in production is $1/3$ and that the labor composition of these economies is unchanged. For each economy, compute the growth rate of TFP.
- (a) A European economy: $g_{Y/L} = .03$, $g_{K/L} = .03$.
 - (b) A Latin American economy: $g_{Y/L} = .02$, $g_{K/L} = .01$.
 - (c) An Asian economy: $g_{Y/L} = .06$, $g_{K/L} = .15$.

6.13 Worked Exercises

Exercise 6: *Putting some numbers into the Romer model.* Consider the basic Romer model studied in this chapter. Suppose the parameters of the model take the following values: $\bar{A}_0 = 100$, $\bar{\ell} = .10$, $\bar{z} = 1/500$, and $\bar{N} = 100$.

- (a) What is the growth rate of per capita output in this economy?
- (b) What is the initial level of per capita output? What is the level of per capita output after 100 years?
- (c) Suppose the research share were to double. How would you answer to the two previous parts of this problem change?

Answer:

- (a) The growth rate of per capita output in the Romer economy is equal to the growth rate of ideas, given by the formula in equation (6.7):

$$\frac{\Delta A_t}{A_t} = \bar{z}L_{at} = \bar{z}\bar{\ell}\bar{N}.$$

With the parameter values given in this problem, this growth rate is $.10 \times \frac{1}{500} \times 100 = .02$, so the economy grows at 2% per year.

- (b) The level of per capita income is given by equation (6.9):

$$y_t = \bar{A}_0(1 - \bar{\ell})(1 + \bar{g})^t,$$

where $\bar{g} \equiv \bar{z}\bar{\ell}\bar{N} = .02$. Substituting in the relevant parameter values, we find that

$$y_0 = 100 \times (1 - .10) = 90,$$

and

$$y_{100} = 100 \times (1 - .10) \times (1.02)^{100} = 652.$$

- (c) If the research shares doubles to 20%, the economy behaves as follows. The growth rate doubles to 4% per year, and the income levels are given by

$$y_0 = 100 \times (1 - .20) = 80,$$

and

$$y_{100} = 100 \times (1 - .20) \times (1.04)^{100} = 4040.$$

The output of the consumption good is lower in the short run because more people are engaged in research. The economy grows faster as a result, however, and incomes in the future are much higher. This exercise is the numerical version of the change considered in the text surrounding Figure 6.4.

6.14 APPENDIX: The Combined Solow-Romer Model

In presenting the Romer model, we made the simplifying assumption that there was no capital in the economy. This is helpful for seeing the insights of the Romer model. However, because the Solow model also helps us answer many questions about economic growth, it is important to understand how to combine the Solow and Romer frameworks.

This appendix shows how the insights of Solow and Romer can be combined in a single model of economic growth. Mathematically, the combination is relatively straightforward. Intuitively, all of the results we have learned from both models continue to hold in the combined model.

6.14.1 Setting up the Combined Model

In order to combine the Romer and Solow models, we start with the Romer model and then add capital back in. The combined model features five equations and five unknowns:

$$Y_t = A_t K_t^{1/3} L_{yt}^{2/3} \quad (6.13)$$

$$\Delta K_t = \bar{s} Y_t - \bar{d} K_t \quad (6.14)$$

$$\Delta A_t = \bar{z} A_t L_{at} \quad (6.15)$$

$$L_{yt} + L_{at} = \bar{N} \quad (6.16)$$

$$L_{at} = \bar{\ell} \bar{N}. \quad (6.17)$$

Our five unknowns are output Y_t , capital K_t , knowledge A_t , workers L_{yt} , and researchers L_{at} .

The first equation above is the production function for output. Notice that it exhibits constant returns to objects — capital and workers. Because of the nonrivalry of ideas, however, it exhibits increasing returns to objects and knowledge together.

The second equation describes the accumulation of capital over time. The change in the capital stock is equal to new investment $\bar{s} Y_t$ less depreciation $\bar{d} K_t$. As in the Solow model, the investment rate, \bar{s} , is an exogenously-given parameter of the model.

The last three equations are all directly imported from the Romer model. New ideas are produced using the existing stock of ideas and researchers. We've left capital out of this production function, but that is only because it makes the model easier to solve. Nothing of substance would change if we instead had capital and researchers combine with ideas to produce new ideas. Equation (6.16) says that the number of workers and researchers sum to equal the total population. And the last equation captures our assumption that a constant fraction of the population, $\bar{\ell}$, works as researchers. This implies that the fraction $1 - \bar{\ell}$ works to produce the output good.

6.14.2 Solving the Combined Model

Suppose we start the economy off at time 0 with \bar{K}_0 units of capital and \bar{A}_0 ideas. How does it evolve over time?

One thing to notice about the combined model is that it is very much like our original Solow model, except that now we have a productivity level A_t that grows over time at a constant rate. In our original Solow model, the productivity level was a constant parameter. A one-time increase in this productivity level produced transition dynamics that led the economy to grow for awhile before settling down at its new steady state.

Now, however, A_t increases continuously over time. In a Solow diagram, this would show up as the $\bar{s}Y$ curve shifting upwards each period, leading the capital stock to increase each period as new investment exceeds depreciation.

This means two things. First, it helps us to understand how capital and output will continue to grow in our combined model. Rather than getting a steady state with a constant level of capital, we will get a balanced growth path, where capital grows at a constant rate. Second, linking the combined model to the Solow diagram suggests that transition dynamics are likely to be important. This suggestion turns out to be correct.

In what follows, we will begin by showing how to solve for the balanced growth path. Later, we take up the issue of transition dynamics.

6.14.3 Long-Run Growth

Inspired by the Romer model, let's look for a balanced growth path — that is, for a situation in our combined model where output, capital, and the stock of ideas all grow at constant rates. The first step in finding this solution is to apply the rules for computing growth rates that we developed back in Chapter ?? to the production function for output. In fact, one of the examples we worked out in that chapter is exactly the problem we have before us now. If you do not understand the derivation in the next paragraph, take a look back at the example in Section ?? of Chapter ?? and the steps are laid out there in detail.

We use two of the rules for computing growth rates: the growth rate of a product is the sum of the growth rates of the terms, and the growth rate of a variable raised to some power is that power times the growth rate of the variable. Applied to the production function for output in equation (6.13), we have

$$g_{Yt} = g_{At} + \frac{1}{3}g_{Kt} + \frac{2}{3}g_{Lyt}, \quad (6.18)$$

where $g_{Yt} \equiv \Delta Y_t / Y_t$, and the other growth rates are defined in a similar way. Notice that g_Y , g_A , and g_K are all endogenous variables; they are just the growth rates of our regular endogenous variables.

Equation (6.18) is really just the growth rate version of the production function. It says that the growth rate of output is the sum of three terms: the growth rate of knowledge, the growth contribution from capital, and the growth contribution from workers. Notice that the growth contributions of capital and workers get weighted by their exponents in the production function, reflecting the diminishing returns to each of these inputs.

To solve for the growth rate of output, we need to know the growth rate of the three terms on the right-hand side of this equation. The growth rate of knowledge, g_A , turns out to be easy to obtain. Just as in the Romer model, it comes directly from dividing the production function for new ideas by the level of knowledge:

$$g_{At} = \frac{\Delta A_t}{A_t} = \bar{z}L_{at} = \bar{z}\bar{\ell}\bar{N}. \quad (6.19)$$

Knowledge grows because researchers invent new ideas. It turns out to be convenient to define $\bar{g} \equiv \bar{z}\bar{\ell}\bar{N}$, just as we did in the Romer model.

We can learn about the second term in equation (6.18) by looking back at the capital accumulation equation, equation (6.14). Dividing that equation by K_t yields

$$g_{Kt} = \frac{\Delta K_t}{K_t} = \bar{s} \frac{Y_t}{K_t} - \bar{d}. \quad (6.20)$$

Notice that this equation still has two endogenous variables on the right side, so it is not yet a solution. But we can still learn something very important from this equation. In particular, ask yourself “What must be true about Y_t and K_t in order for g_K to be constant over time?” Notice that all of the other terms in equation (6.20) are constant along a balanced growth path, so that Y_t/K_t must be constant as well. But the only way this ratio can be constant is if Y_t and K_t grow at the same rate. For example, if Y_t grew faster than K_t , then the ratio would grow over time, causing g_K to increase. This means we must have $g_K^* = g_Y^*$, where we’ve used the superscript asterisk (*) to denote the fact that these variables are evaluated along a balanced growth path. At this point, of course, we do not know the values of either g_Y^* or g_K^* , but now if we can figure out one of them, we will know the other.

Finally, take a look at the last term in equation (6.18). This is the growth rate of the number of workers. We’ve assumed the number of workers is a constant fraction of the population, and we’ve assumed the population itself is constant. This means that the growth rate of the number of workers must be equal to zero: $g_{Lyt} = 0$.

Now we are ready to plug our three results back into the growth rate version of the production function. In particular, we have $g_{At} = \bar{z}\bar{\ell}\bar{N} \equiv \bar{g}$, $g_K^* = g_Y^*$, and $g_{Lyt} = 0$. Substituting these three results into equation (6.18) and evaluating that expression along a balanced growth path yields

$$g_Y^* = \bar{g} + \frac{1}{3}g_Y^* + \frac{2}{3} \cdot 0.$$

Notice that this equation just involves a single endogenous variable, g_Y^* , so we can

solve this equation for g_Y^* to find¹⁹

$$g_Y^* = \frac{3}{2}\bar{g} = \frac{3}{2}\bar{z}\bar{\ell}\bar{N}. \quad (6.21)$$

This equation pins down the growth rate of output — and the growth rate of per capita output since there is no population growth — in the long-run of the combined model. It is interesting to compare this solution to what we found in the Romer model. In equation (6.9), we showed that the growth rate of per capita income in the Romer model was exactly equal to \bar{g} . In the combined model, growth in the long run is even faster, at $3/2 \cdot \bar{g}$. Why the difference?

The answer must, of course, be related to capital accumulation, since that is the only real difference between the Romer model and our combined model. Indeed, recall from our Solow diagram what happens when productivity increases in the original Solow model. An increase in productivity causes the level of capital to increase. So output rises for *two* reasons: there is a direct effect from the increase in productivity itself, but then there is an indirect effect because the productivity increase leads to a higher capital stock, which in turn leads to an even higher level of output. This is exactly what happens in our combined model. There is a direct effect of growth in knowledge on output growth; this was obvious back in equation (6.18). But then the growth in output leads to capital accumulation, which in turn leads to more output growth.

So while capital cannot itself serve as an engine of economic growth, it helps to amplify the underlying growth in knowledge. Long-run growth in per capita income is therefore higher in the combined model than in the Romer model.

6.14.4 Per Capita Output

Now that we know the growth rate of output in the combined model, we can also solve for the *level* of per capita output along a balanced growth path. The process of getting this solution is exactly the same as in the original Solow model in Chapter ??.

First, let's get an equation for the capital stock. Look back at equation (6.20) above, and recall that $g_K^* = g_Y^*$ along a balanced growth path. That equation can be solved

¹⁹In particular, subtract $1/3 \cdot g_Y^*$ from both sides to find $2/3 \cdot g_Y^* = \bar{g}$ and then multiply both sides by $3/2$ to get the solution.

for the capital-output ratio along a balanced growth path:

$$\frac{K_t^*}{Y_t^*} = \frac{\bar{s}}{g_Y^* + \bar{d}}. \quad (6.22)$$

This equation says that the capital-output ratio is proportional to the investment rate, \bar{s} , along a balanced growth path. A higher rate of investment will make the economy more capital-intensive, in the sense that the capital-output ratio will be higher.

If we view this solution for the capital-output ratio as an equation giving K_t^* as a function of Y_t^* , we can substitute it back into the production function, i.e. back into equation (6.13), and solve to find²⁰

$$y_t^* \equiv \frac{Y_t^*}{\bar{N}} = \left(\frac{\bar{s}}{g_Y^* + \bar{d}} \right)^{1/2} A_t^{*3/2} (1 - \bar{\ell}). \quad (6.23)$$

This equation can be compared to our solutions for the Romer model, in equation (6.9) above, and for the Solow model, in equation (??) in Chapter ???. As in the Romer model, per capita income depends on the stock of knowledge. Because of non-rivalry, a new idea raises the income of every person in the economy. (By the way, we can solve this equation further by noting that the stock of ideas A_t^* is given by the same equation as in the Romer model, equation (6.8).) Notice that growth in A_t leads to sustained growth in per capita income along the balanced growth path.

The equation also has elements of the Solow model. For example, per capita income depends on the square root of the investment rate. A higher investment rate raises the *level* of per capita income along a balanced growth path; this result is discussed further below in the context of transition dynamics.

²⁰Here is the algebra to get the solution. First, the substitution for K_t^* from equation (6.22) gives

$$Y_t^* = A_t \left(\frac{\bar{s}}{g_Y^* + \bar{d}} \cdot Y_t^* \right)^{1/3} L_{yt}^{2/3}.$$

Collecting the Y_t terms on the left side,

$$Y_t^{*2/3} = A_t \left(\frac{\bar{s}}{g_Y^* + \bar{d}} \right)^{1/3} L_{yt}^{2/3}.$$

Finally, raise both sides to the power 3/2 and divide by \bar{N} to get the equation in the main text.

6.14.5 Transition Dynamics

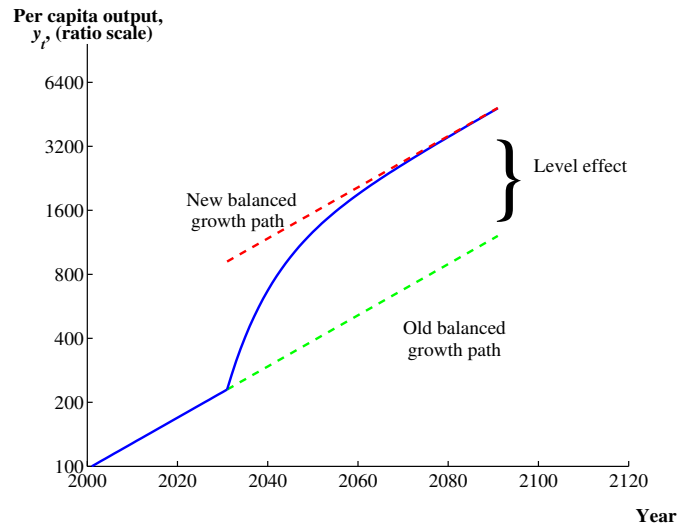
What in the original Solow model led to the presence of transition dynamics? This is an important question, and you should make sure you know the answer to it. The answer is diminishing returns to capital. As capital accumulates, each additional increment to the capital stock raises the level of output — and investment — by less and less, causing the growth rate to decline as the economy approaches its steady state from below.

What about the combined Romer and Solow model? Will it exhibit similar transition dynamics? The answer is yes, and for the same reason. The production function still exhibits diminishing returns to capital, so the principle of transition dynamics applies in this richer model as well. In the following paragraphs, we discuss the transition dynamics of the combined model without the mathematics. Because the math is missing, you may not be able to understand exactly how and why the transition dynamics look the way they do. However, at this point it is sufficient for you to follow the intuition behind the basic result.

In the original Solow model, the principle of transition dynamics said that the further below its steady state an economy was, the faster it would grow. In the combined Solow-Romer model, we must change this statement. There is no longer a steady state, and instead, the economy grows at a constant rate in the long run. Nevertheless, a similar statement still applies. For the combined model, the *principle of transition dynamics* can be stated as

The further below its balanced growth path an economy is (in percentage terms), the faster the economy will grow. Similarly, the further above its balanced growth path an economy is, the slower it will grow.

To understand our new version of this principle, it is helpful to consider an example. Suppose the economy starts out on its balanced growth path, but then the investment rate \bar{s} is increased to a permanently higher value. How does the economy evolve over time? It should be clear that immediately following the change, the economy is below its balanced growth path. From equation (6.23), the increase in the investment rate means that the balanced growth path level of income is now higher. Since current income is unchanged, the economy is now below its balanced growth path. According

Figure 6.5: Output over Time after a Permanent Increase in \bar{s} 

The graph shows the evolution of per capita output, y_t . The economy begins on a balanced growth path. Then, in year 2030, there is a permanent increase in the investment rate, \bar{s} . This raises the balanced growth path. Since the economy is now below its new balanced growth path, the principle of transition dynamics says that the economy will grow rapidly. Notice that y_t is plotted on a ratio scale, so the slope of the path is the growth rate.

to the principle of transition dynamics, we should expect the economy to grow rapidly to “catch up” to this path.

This example is shown graphically in Figure 6.5. A key thing to notice about this graph is that per capita output is plotted on a ratio scale. Recall that this means that the slope of the output path is related to the growth rate of y_t . For example, before the increase in the investment rate, y_t is growing at a constant rate: the path is a straight line. After the increase in year 2030, the growth rate rises immediately — the slope of the path increases sharply. Over time, the growth rate declines until eventually it has the same slope as the original path. That is, the growth rate returns to \bar{g} , which does not depend on \bar{s} . Notice that the level of per capita output is permanently higher as a result of the increase in the investment rate, but the growth rate is unchanged. This is sometimes called a long-run “level effect.” Overall, this graph shows the principle of transition dynamics at work in the combined model.

What changes in the combined model lead to transition dynamics? The answer is that changing any parameter of the model will create transition dynamics. Why? The parameters are \bar{s} , \bar{d} , \bar{z} , \bar{N} , $\bar{\ell}$, \bar{K}_0 , and \bar{A}_0 . If you look back at the solution of the combined model in equations (6.21) and (6.23), you will see that all of these parameters affect either (i) the level of output along the balanced growth path, or (ii) the level of current output. In particular, changes in each of these parameter values will create a gap between current output and the balanced growth path, just like the one we saw in Figure 6.5. Once this gap is created, the principle of transition dynamics takes over and the economy grows to close the gap.

The reason it is important to recognize that the combined model still features transition dynamics is because the principle of transition dynamics was the key to understanding differences in growth rates across countries in the Solow model. What we see is that this explanation continues to apply in the combined model. Now we have a theory of long-run growth driven by the discovery of new ideas throughout the world *and* a theory of differences in growth rates across countries based on transition dynamics. Our model predicts that in the long-run, all countries should grow at the same rate, given by \bar{g} , the growth rate of world knowledge. However, over any given period of time, one may observe differences in growth rates across countries based on the fact that not all countries have reached their balanced growth paths. Changes in policies that change the parameters of the model — like the investment rate — can lead to differences in growth rates over long periods of time.

6.14.6 More Exercises

1. What is the principle of transition dynamics in the combined Solow-Romer model?
2. Growth in the combined Solow-Romer model is faster than growth in the Romer model. In what sense is this true? Why is it true?
3. *Balanced growth.* Suppose we observe the following growth rates in various economies. Discuss whether or not each economy is on its balanced growth path.
 - (a) A European economy: $g_{Y/L} = .03$, $g_{K/L} = .03$.

(b) A Latin American economy: $g_{Y/L} = .02$, $g_{K/L} = .01$.

(c) An Asian economy: $g_{Y/L} = .06$, $g_{K/L} = .15$.

4. *Transition dynamics in the combined Solow-Romer model.* Consider the combined model studied in this appendix. Suppose the economy begins on a balanced growth path in the year 2000. Then in the year 2030, the depreciation rate \bar{d} rises permanently to the higher level \bar{d}' .

(a) Graph the behavior of per capita output over time, using a ratio scale.

(b) Explain what happens to the growth rate of per capita output over time and why.

5. *The Combined Romer-Solow model.* Consider the basic combined model that we studied in this appendix. Let's make one change to that model: let the production function for output be $Y_t = A_t K_t^{1/4} L_{yt}^{3/4}$. That is, we've reduced the exponent on capital and raised it on labor, to preserve constant returns in objects.

(a) Solve for the growth rate of output per capita along a balanced growth path. Explain why it is different from the model considered in the chapter.

(b) (hard) Solve for the level of per capita output along a balanced growth path. Explain how and why this solution differs from what we found in the chapter.

6. *The Combined Solow-Romer model (hard).* Consider again the original combined model. Let's make one change to that model: let the production function for output be $Y_t = A_t K_t^\alpha L_{yt}^{1-\alpha}$. That is, we've made the exponent on capital a parameter (α) rather than keeping it as a specific number. Notice that this affects the exponent on labor as well, in order to preserve constant returns to objects in production.

(a) Solve for the growth rate of output per capita along a balanced growth path. Explain how it relates to the solution of the model considered in the chapter.

(b) Solve for the level of per capita output along a balanced growth path. Explain how it relates to the solution of the model considered in the chapter.

- (c) The formula for a geometric series is $1 + \alpha + \alpha^2 + \alpha^3 + \dots = \frac{1}{1-\alpha}$ if α is some number between zero and one. How and why is this formula related to your answers to the two previous parts of this problem? Hint: Think about how an increase in output today affects capital in the future.